THEORETICAL STUDY OF TURBULENT SALINE DIFFUSION
IN A CONDUIT IN CONTINUOUS SERVICE

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ABSTRACT

The distribution of concentration in a point-type injection is obtained by solving the diffusion equation, after which the effect of hypothetical calculations on the solutions arrived at is examined.

In a previous paper, we achieved solutions of the diffusion equation. Supposing now that $c(z, \theta, 0)$ is the concentration at $\xi = 0$, the concentration on the abscissa ξ will be given by the equation

$$c(z, \theta, \xi) = \sum_{m=0}^{m=\infty} (b_{mp} \cos m\,\theta + c_{mp} \sin m\,\theta) \sum_{p=0}^{p=\infty} f_{m,p}(z)\, e^{-\beta_{m,p}\xi} \qquad (9)$$

the coefficients b_{mp} and c_{mp} being calculated according to the classical methods applicable to orthogonal functions.

It is possible, for example, to start with the injection section and to consider the brine jet to be the equivalent of a semi-point discharge.

It should be noted that the discharge of salt is usually accomplished by a jet that causes a different turbulent diffusion from that existing in a conduit, and a higher one in any case. This procedure underestimates, then, mixture by diffusion.

We shall calculate in this way the concentration resulting from a point discharge at the coordinates $z = z_0$ and $\theta = 0$.

The following are the first useful terms of the development for numerical values of $k = 0.4$ and $n = 21$ in order of increasing damping:

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[1]Presented at the meeting, 11 March 1957.

$$(14)$$

$$z(1-z)f'' + (2-3z)f' + s\frac{u(z)}{U}f = 0.$$

or in our case

$$z(1-z)f'' + (2-3z)f' + \sigma(1-z^p)f = 0, \qquad \sigma = s\frac{p+2}{p}. \qquad (15)$$

The coefficients of the development of finite f for z = 0

$$f(z) = \sum_{n=0}^{n=\infty} a_n Z^n \qquad (16)$$

establish the recurrence equation

$$a_{n+1} = \frac{[n(n+2) - \sigma]a_n + \sigma \cdot a_{n-p}}{(n+1)(n+2)}. \qquad (17)$$

Generally speaking, the series (16) does not converge for z = 1, except for the characteristic values of σ. These characteristic values can be obtained by the use of the continuity equation:

$$\int_0^1 f(z, \sigma) z(1 - z^p)\, dz = 0. \qquad (18)$$

To establish this property, we multiply the terms of (15) by z and integrate from 0 to 1 as follows:

$$[f'z^2(1-z)]_0^1 + \int_0^1 f(z, \sigma) z(1 - z^p)\, dz = 0. \qquad (19)$$

f is finite at point z = 0, but it has generally a logarithmic point at z = 1, the derivative f' being, therefore, of the form a/(1-z), and the term between parentheses not being zero. For the characteristic values of σ, f remains finite, the term between parentheses is zero, and equation (18) is satisfactorily established. The integration, term by term, of f according to (18) gives the series

$$F(\sigma) = \sum_{n=0}^{n=\infty} a_n \frac{p}{(n+2)(n+p+2)}. \qquad (20)$$

The convergence is gradual. We have calculated the first root, $\sigma = 4.166$, to 17 places, for $p = 6$. The corresponding value of s is 3.1245 instead of s = 3 in the case of a uniform distribution of velocities.

Thus the assumption that there is a uniform distribution of velocities leads to an acceptable approximation.

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